

SECTION 1

INTRODUCTION

For determining the trajectory of a spacecraft, computed values of observed quantities are fit to the observables by varying the values of the model parameters. The estimated values of these so-called solve-for parameters determine the trajectory of the spacecraft. This report documents the current formulation for the observed and computed values of the observables and the corresponding partial derivatives of the computed observables with respect to the solve-for parameters. This formulation is used in program Regres of the Orbit Determination Program (ODP) of the Jet Propulsion Laboratory. This third-generation program has been used to determine spacecraft trajectories for lunar and planetary missions since 1968. Recently, it has also been used to determine the orbits of Earth satellites.

The last external report which documented the Regres formulation was Moyer (1971). The scope of that report was the formulation of the entire ODP. This report documents the complete formulation of program Regres of the ODP and the relativistic terms of the formulation of program PV, which generates the spacecraft trajectory and the corresponding partial derivatives with respect to the estimable parameters. Thus, this document contains all of the relativistic terms that affect the computed values of observed quantities. The complete formulation of program PV will eventually be documented by Richard F. Sunseri, the programmer/analyst for that program. The user's guide for the ODP is given in *DPTRAJ-ODP User's Reference Manual* (2000).

All of the observables can be placed into the following broad categories: doppler, range, spacecraft and quasar very long baseline interferometry (VLBI), and angular observables. They are described in detail in Section 13. The model parameters whose values can be estimated can be placed into the following categories:

- (a) Dynamic parameters that determine the spacecraft trajectory,

SECTION 1

- (b) Station location parameters that determine the Earth-fixed locations of the tracking stations,
- (c) Earth orientation parameters that determine the space-fixed orientation of the Earth,
- (d) Reference parameters that determine the relative positions of the celestial bodies of the Solar System,
- (e) Quadratic coefficients of corrections to atomic time at the spacecraft and tracking stations,
- (f) Quadratic coefficients of the correction to the spacecraft transmitter frequency (when it is the transmitter),
- (g) Range biases,
- (h) Parameters of the Earth's troposphere and ionosphere,
- (i) The relativity parameters β and γ ,
- (j) The right ascensions and declinations of quasars.

Those parameters, such as range biases, that affect the computed values of the observables but not the position vectors of the participants (the spacecraft and the tracking stations) are referred to as observational parameters.

There are two variations of the formulations used in programs PV and Regres of the ODP. One of these is the original formulation which is referred to the Solar-System barycentric relativistic frame of reference. It applies for a spacecraft anywhere in the Solar System. The alternate formulation is referred to the local geocentric relativistic frame of reference. It applies for a spacecraft near the Earth, such as an Earth orbiter. Note that lunar missions must be analyzed in the Solar-System barycentric frame of reference.

The errors in the computed values of range and doppler observables due to neglected terms in the formulation for computing them are less than 0.2 m

(one-way) and 10^{-6} m/s per astronomical unit (AU) of range to the spacecraft. These figures assume two-way data (the receiving station on Earth is the transmitting station). Also, they do not account for errors in input items, such as the planetary and spacecraft ephemerides, precession and nutation models, and tracking station locations.

Section 2 discusses time scales and the calculation of time differences. This material is presented first because time is discussed in all of the other sections of this report. The planetary and satellite ephemerides and the quantities interpolated from them are described in Section 3. Section 4 presents the equations used in program PV for the acceleration of the spacecraft due to gravity only (Newtonian and relativistic terms) in the Solar-System barycentric and local geocentric frames of reference. Section 5 gives the extensive formulation for the geocentric space-fixed position, velocity, and acceleration vectors of a fixed tracking station on Earth. The formulation for the space-fixed position, velocity, and acceleration vectors of a landed spacecraft on one of the celestial bodies of the Solar System is given in Section 6. Section 7 gives the four algorithms used for calculating the difference between coordinate time of general relativity and atomic time at the transmission or reception time at a tracking station on Earth or an Earth satellite. Section 8 gives the light-time equation and the algorithm for the spacecraft light-time solution. It also gives the corresponding quantities for the quasar light-time solution used in calculating the computed values of quasar VLBI observables. The formulation used to compute the auxiliary angles is given in Section 9. The calculation of antenna, tropospheric, and charged-particle corrections is described in Section 10. Section 11 describes how precision range (round-trip or one-way light times) and quasar delays are calculated from quantities computed in Sections 7 to 10. The partial derivatives of the computed precision ranges and quasar delays with respect to the solve-for parameters are given in Section 12. Section 13 gives the formulations for the observed and computed values of the various types of doppler, range, VLBI, and angular observables, and the equations for calculating media corrections for the computed values of the observables and partial derivatives of the computed values of the observables with respect to the solve-for parameters. The Orbit Data Editor (ODE) obtains the observed quantities from the tracking stations and

SECTION 1

converts them to the “observables” which are used in program Regres, using the formulations given in Section 13. The references are given in Section 14. Acronyms used throughout this document are given in Section 15.